Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) Determine \( k \) such that \( 4(k - 3) + 5 = 7(3k - 1) \).

2) If a line \( l \) is perpendicular to the line \( 3x + 5y = 6 \), what is the slope of \( l \)?

3) The line \( ax + by + 6 = 0 \) passes through the points (-5,2) and (4,1). Determine \( a \) and \( b \).

4) Find all real numbers \( x \) satisfying \( |2x - 3| < 7 \).

5) If \( x = 55 \) and \( y = 150 \), find \( z \).

6) A change box contains $3.00 in dimes and nothing else. A certain number of dimes are removed and then replaced by an equal number of quarters, with the result that the box now contains $4.20. How many dimes are removed?

7) Suppose that toothpicks are used to build the figure below. If the pattern were continued, how many toothpicks would be required to build the 100th figure?

8) If \( f(x) = x^2 - \frac{2}{x} \), find \( f\left(\frac{2}{x}\right) \).

9) A ladder 5m. long is leaning against a building. The angle formed by the ladder and the ground is 60°. How far from the building is the foot of the ladder?

10) Solve for \( x \): \( \log_5(2x - 1) = 2 \).
11) Find all values \( x \) such that \( x(2x - 5) = (x + 3)x \).

12) In triangle \( AOB \), \( AO = AB \), and the measure of \( \angle A \) is \( 100^\circ \). The circle centered at \( O \) has a radius of \( 6 \). Find the area of the shaded sector.

13) If \( \sec \theta = - \frac{5}{3} \) and \( \sin \theta < 0 \), find \( \tan \theta \).

14) Solve for \( x \): \( \frac{3x + 5}{2x - 3} = y \).

15) A rhombus has a side length of \( 4 \) inches and an angle of \( 60^\circ \). What is the area of the rhombus?

16) In the circle centered at \( O \), the measure of \( \angle AOB \) is \( 120^\circ \), and the length of the minor arc \( \widehat{AB} \) is \( 6\pi \) meters. Determine the radius of the circle.

17) Find the center and radius of the circle with equation \( x^2 + y^2 - 6y = 16 \).

18) Shannon spent \$50 on tapes, and then spent \( \frac{2}{5} \) of her remaining money on books; after that, she used \( \frac{1}{3} \) of the remaining amount to buy gifts. She was left with \$48. How much money did Shannon have initially?

19) If the operation \( \Box \) is defined \( x \Box y = 2x + 3y \) find \( (4 \Box 5) \Box 6 \).

20) Suppose \( A(x) \) denotes the area of a triangle constructed as follows. A yardstick is cut into 2 pieces of lengths \( x \) and \( 3 - x \), and these pieces, together with a 1–foot stick, are used to form the triangle. Determine the domain and range of the function \( A \). (Note the maximum possible area occurs when the triangle is isosceles)
1) Solve for \( x \): \( \sqrt{x + 3} = 2 + \sqrt{x - 5} \)

2) For a certain event, 812 tickets were sold totaling $1912. If students paid $2 per ticket and non-students paid $3 per ticket, how many student tickets were sold?

3) In triangle \( ABC \), \( m\angle A = 50^\circ \), \( m\angle C = 80^\circ \), \( AC = 7x + 8 \), and \( BC = 38 - 3x \). Determine the value \( x \).

4) Give the radian measure of a 330° angle.

5) Determine all roots of the equation \( x^3 - 4x^2 + x + 6 = 0 \).

6) The center of a circle lies in the second quadrant and is 1 unit from the \( y \)-axis and 2 units from the \( x \)-axis. If the circle is tangent to the \( y \)-axis, find the equation of the circle.

7) If the angle \( \theta \) is acute and \( \sin \theta = a \), express \( \sin 2\theta \) in terms of \( a \).

8) Solve for \( x \): \( 4^{2x + 1} = 8 \).

9) Suppose \( f \) is a function with graph as shown. List all of the statements (a) - (e) which are true.

   (a) \( f \) is increasing on \([-1,1]\)
   (b) \( f(0)f(2) > 0 \)
   (c) \( f(0) + f(2) > 0 \)
   (d) \( f(f(-1)) = 1 \)
   (e) \( f \) has only integer roots

10) In triangle \( ABC \), \( AE = 6 \), \( CE = 10 \), \( AB = 14 \), and \( DE \parallel BC \). Find \( AD \).
11) Milton has $35\frac{1}{2}$ yd. of ribbon to make bows. Each bow requires $\frac{3}{8}$ yd. of ribbon.
   (a) How many bows can be made?
   (b) How much ribbon will be left over?

12) A parabola has a vertical axis and passes through the points (-1,0), (5,0), and (1,8). Find the equation of the parabola.

13) Determine the period of the function $f(x) = \sin (6x - \pi)$.

14) Find all $x$ in the interval $[0,2\pi)$ such that $|\cos x| \leq \frac{\sqrt{2}}{2}$.

15) Determine $A$ and $B$ such that $rac{2x - 2}{x^2 + 7x + 10} = \frac{A}{x + 2} + \frac{B}{x + 5}$.

16) A rectangular box is packed with six cylindrical soda cans, as shown. Find the ratio of total volume of the cans to the volume of the box.

17) Suppose each interior angle of a regular polygon measures $174^\circ$. How many sides does the polygon have?

18) A school has 1500 students. Each student takes 6 classes. Each teacher teaches 5 classes. Each class has 30 students and 1 teacher. How many teachers does the school have?

19) A multiple choice test consists of 5 questions, each with 3 possible answers. If a student guesses randomly on each question, what is the probability that she answers all questions correctly?

20) Suppose the fare for a taxi is $2.50 for the first $\frac{1}{2}$ mile, plus 50 cents for each additional $\frac{1}{4}$ mile. Given that $n$ denotes a positive integer, and $f(n)$ denotes the fare in dollars for an $n$–mile ride, give the formula for $f(n)$. 