1. Find a rational number $x$, such that

$$2\pi < x < \sqrt{40}$$

**Ans:** (answers may vary)  Since $2\pi \approx 6.28$ and $\sqrt{40} \approx 6.32$, $x = 6.3$

2. Find an irrational number, $x$, such that

$$\frac{\pi}{2} < x < \frac{8}{5}$$

**Ans:** (answers may vary)  $x = \frac{1}{2} (2\pi + \frac{8}{5})$ or $\frac{101}{200} \pi$ or $\frac{5}{\pi}$

3. Suppose that we have 0.202002000200002000002000002... . If we continue in this way inserting one more zero between each successive pair of twos, is this number rational or irrational? Explain why.

**Ans:** irrational because it is non-terminating and nonrepeating

4. What is the average of 0.09 and $\frac{3}{5} \pi$?

**Ans:**  $x = \frac{1}{2} (\frac{3}{5} \pi + 0.09)$

5. Arrange the following real numbers in increasing order —

$$\frac{-1}{\pi} \approx -.318 \quad 10 \sqrt{10} \approx 31.62 \quad \frac{-1}{10} \pi \approx -.314 \quad \frac{3}{8} = .375 \quad 0.31 \approx .313$$

**Ans:**  $\frac{-1}{\pi} < \frac{-1}{10} \pi < 0.31 < \frac{3}{8} < 10 \sqrt{10}$
6. Explain why the set of real numbers between $-1$ and $+1$ is (or is not) closed under multiplication?

Ans: Any number between $-1$ and $+1$ is a proper fraction, and when you multiply a proper fraction by a proper fraction, the product has a smaller absolute value less than any of its factors, thus putting the product between $-1$ and $+1$. Therefore, this set is closed under multiplication.

7. In 1844 the French mathematician Joseph Liouville (1809-1882) proved nonalgebraic or transcendental numbers existed. Liouville’s number can be calculated from the infinite series below.

\[
\frac{1}{10^1} + \frac{1}{10^2} + \frac{1}{10^3} + \frac{1}{10^4} + \frac{1}{10^5} \ldots
\]

Show an approximation (as a decimal fraction) of the value of Liouville’s number by carrying out the above addition to the fourth term.

\[
\frac{1}{10^1} = \frac{1}{10} = 0.1
\]

\[
\frac{1}{10^2} = \frac{1}{100} = 0.01
\]

Ans: \[
\frac{1}{10^3} = \frac{1}{1000} = 0.000001
\]

\[
\frac{1}{10^4} = \frac{1}{10000} = 0.00000000000000000000000000000001
\]

so, if we sum these terms we obtain 0.11000100000000000000000000000001

8. What can be said about $n(\mathbb{R})$ compared to $n(\mathbb{N})$?

Ans: $n(\mathbb{R}) > n(\mathbb{N})$
9. Use the following mnemonic to write an approximate value for \( \pi \).

\[
\begin{align*}
\text{Ans:} & \quad 3 \quad 1 \quad 4 \quad 1 \quad 5 \quad 9 \\
& \quad \text{Sir, I bear a rhyme excelling} \\
& \quad 2 \quad 6 \quad 5 \quad 3 \quad 5 \quad 8 \\
& \quad \text{In mystic force and magic spelling} \\
& \quad 9 \quad 7 \quad 9 \\
& \quad \text{Celestial sprites elucidate} \\
& \quad 3 \quad 2 \quad 3 \quad 8 \quad 4 \quad 6 \\
& \quad \text{All my own striving can't relate.} \\
& \quad 2 \quad 6 \quad 4 \quad 3 \quad 3 \quad 8 \\
& \quad \text{Or locate they who can cogitate} \\
& \quad 3 \quad 2 \quad 7 \quad 9 \quad 5 \\
& \quad \text{And so finally terminate. Finis}
\end{align*}
\]

Therefore, \( \pi \approx 3.1415926535897932384626433832795 \)

10. The value of the continued fraction below is an approximation for \( \sqrt{2} \). What is this approximate value?

\[
\begin{align*}
1 \quad + \quad & \frac{1}{2 \quad + \quad \frac{1}{2 \quad + \quad \frac{1}{2 \quad + \quad \frac{1}{2}}}}
\end{align*}
\]

\[
\text{Ans:} \quad 1 \frac{19}{45} \quad \text{or} \quad 1.4 \quad \text{or if decimal fractions were used consistently} \quad 1.414285714