1. The current in a given circuit is given by $\frac{dq}{dt} = e^{-2t}\cos t$. Find an expression for the amount of charge which passes a given point in the circuit as a function of time, if $q(0)=0$.

2. A tree has been transplanted and after $x$ years is growing at a rate of $\frac{x^2 + 2x + 3}{x^2 + 2x + 1}$ meters per year. After 2 years it has reached a height of 5 meters. How tall was it when it was transplanted?

3. Under certain conditions the velocity as a function of time is given by $v = \frac{2}{\sqrt{t^2 + 100}}$. Determine the displacement as a function of time if $s = 6$ when $t = 0$.

4. Find the general solution to the differential equation $\frac{dy}{dt} = 5 - 2\ln t$.

5. Consider a particular growth model defined by $\frac{dP}{dt} = .2P(1 - \frac{P}{10})$ lb/hr; $P(0) = .5$ lb

   a) Obtain the formula for $P(t)$.
   b) How large will $P$ be after 3 hours?
   c) When will $P$ reach one-half the carrying capacity - that is, for which $t$ is $P = 5$ lb?

6. In a bimolecular reaction, the rate of reduction is given by $\frac{dy}{dt} = \frac{t - t + 1}{t^2(t - 1)}$

   Express $y$ as a function of $t$.

Note: You must illustrate the appropriate integration techniques to receive full credit for any of the above problems. You will receive partial credit for any problems solved by using a table of integrals.
1. It is necessary to evaluate \( f(x) = \int \frac{dx}{\sin x + \tan x} \) to monitor certain cardiac arrhythmias. If \( f\left(\frac{\pi}{2}\right) = \frac{1}{2} \), find \( f(x) \).

2. Show, either by an induction proof or by the method of finite differences, that

\[
\sum_{k=1}^{n} k^3 = \left(\frac{1}{2} n(n+1)\right)^2
\]

3. Supply an induction proof for \( 2^n < n! \) for \( n \geq 4 \).