1. Given a group of people who know each other, define the relation characterized by the linguistic expression *a friend of*. Which properties (symmetry, transitivity, etc.) does this relation posses?

2. Order the equivalence relation obtained by the grade matrix handout by the relation of *partition refinement*. Show that the relation is a partial ordering.

3. Determine the matrix and diagram representations of the compatibility relations on the set of students that are based on the following definitions of compatibility: Two students are compatible if they do not differ in more than two of the four characteristics

4. Let \( A = \{1,2,3\} \). Which of the following binary relations on \( A \times A \) is reflexive, symmetric, transitive, or antisymmetric?
   
   a) \( R_1 = \{ <x, y> | x < y \} \)
   
   b) \( R_2 = \{ <x, y> | 2x = y \} \)
   
   c) \( R_3 = \{ <x, y> | x = y - 1 \} \)
   
   d) \( R_4 = \{ <0, 0>, <0, 1>, <1, 0>, <1, 1>, <1, 2>, <2, 2>, <0, 2>, <3, 3> \} \)

5. Determine the inverse relation for each of the relations in problem #4 and categorize the relations via \( R, S, A, T \).

6. For each of the following functions, determine the supremum and infimum, as well as the maximum and minimum (if they exist).
   
   a) \( f(x) = 5 \)
   
   b) \( f(x) = 5 - x \), \( x \in [1, 5) \)
   
   c) \( f(x) = \sin x \), \( x \in [0, 2\pi] \)
   
   d) \( f(x) = \begin{cases} x & \text{when } x \in [0, 1) \\ 1 - x & \text{when } x \in [1, 2] \\ 0 & \text{otherwise} \end{cases} \)
   
   e) \( f(x) = \frac{x}{1 + x} \), \( x \in (0, 10) \)
   
   f) \( f(x) = \frac{\sin x}{x} \), \( x \in \mathbb{R} \)
7. For each of the following relations in two-dimensional Euclidian space, determine its one-dimensional projections, its cylindric extensions, and the intersection of the cylindric extensions. In each case, show also a geometric interpretation of the relation and the other relations derived from it.

a) \( R = \{ <x, y> | x^2 + y^2 \leq 1 \} \)

b) \( R = \{ <x, y> | x \in [-1, 1], y \in [-1, 1] \} \)

c) \( R = \{ <x, y> | |x| + |y| \leq 1 \} \)

d) \( R = \{ <x, y> | x^2 + 2y^2 \leq 1 \} \)

8. A compatibility relation on \{ x_1, \ldots, x_6 \} given by the conventional array:

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  1 1 1
  1 0 1
1 0 0 1 1
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a) Draw the graph.

b) Give the complete cover.

9. The Venn diagram represents the three elements of a complete cover. Draw the graph of the compatibility relation.

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\text{Complete Cover} = \{ C_1, C_2, C_3 \}
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10. Draw a graph of the relation on the set of all 3-bit strings \( R = \{ <x, y> | x - y \text{ is an even number of bits} \} \) (e.g. 001 - 010 = 2 bits). What kind of relation is this?